

## Modified Imperialistic Competitive Algorithm in Hopfield Neural Network for Boolean Three Satisfiability Logic Mining

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### ABSTRACT

Artificial neural networks (ANNs) are actively utilized by researchers due to their extensive capability during the training process of the networks. The intricate training stages of many ANNs provide a powerful mechanism in solving various optimization or classification tasks. The integration of an ANN with a robust training algorithm is the supreme model to outperform the existing framework. Therefore, this work presented the inclusion of three satisfiability Boolean logic in the Hopfield neural network (HNN) with a sturdy evolutionary algorithm inspired by the Imperialist Competitive Algorithm (ICA). In general, ICA stands out from other metaheuristics as it is inspired by the policy of extending the power and rule of a government/country beyond its own borders. Existing models that incorporate standalone HNN are projected as non-versatile frameworks as

it fundamentally employs random search in its training stage. The main purpose of this work was to conduct a comprehensive comparison of the proposed model by using two real data sets with an elementary HNN with exhaustive search (ES) versus a HNN with a standard evolutionary algorithm, namely- the genetic algorithm (GA). The performance evaluation of the proposed model was analyzed by computing plausible errors, such as root mean square error (RMSE), mean absolute error (MAE),

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global minima ratio ( $R_m$ ), computational time ( $CT$ ) and accuracy ( $Q$ ). The computational simulations were carried out by operating the different numbers of neurons in order to validate the efficiency of the proposed model in the training stage. Based on the simulations, the proposed model was found to execute the best performance in terms of attaining small errors and efficient computational time compared to other existing models.

*Keywords: 3-satisfiability, Hopfield neural network, imperialist competitive algorithm, logic mining*

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## INTRODUCTION

The inception of artificial neural networks (ANNs) has initiated a variety of capable models, which act as a useful tool in solving specific tasks such as classification, prediction, and pattern recognition (Ghaleini et al., 2019). Many of the recent developments have assembled different takes in refining the existing ANN models, specifically by integrating them with proficient searching techniques in order to intensify the quality of their standalone framework. In general, ANN possesses comprehensive structure of training and testing stages, thus emerging as one of the most efficient tools in finding patterns and extracting information to solve real-life applications. They are implemented in tasks such as solar radiation forecasting (Benali et al., 2019), risk analysis (Shi et al., 2019), fault detection (Dybkowski & Klimkowski, 2019), and quantitative analysis (Li et al., 2019a). Accordingly, ANNs can be described in many forms; one of them is the feedback-inducing recurrent networks. In particular, Hopfield neural network (HNN) is a recurrent neural network resembling the operations of human memory (Hureira & Vartanian, 2019). Proposed by Hopfield and Tank in 1985, its ability to manage nonlinear patterns by its training and testing capabilities is especially useful for interpreting complex real-life problems. In recent years, HNN has been widely used by many researchers as it has a deliberately sturdy component of content addressable memory (CAM) (Kong et al., 2019) and emits a degree of convergence by utilizing an energy function (Kasihmuddin et al., 2019). However, the fundamental HNN employs dated heuristics in its training stage, namely the exhaustive search (ES). Nievergelt (2000) had discovered that ES was not considered as a robust search technique as it exerted a random search mechanism, which increased the tendency of overfitting and showed the lack of variations (Lim & Bang, 2010). As such, Mansor et al. (2019) had proposed the incorporation of Elliot Hopfield Neural Network (EHNN) with a modified artificial immune system algorithm (AIS), thus ameliorating the performance of the HNN elementary model. Other than that, Genitha and Vani (2019) had proposed an integrated framework of modified genetic algorithm (GA) with the HNN approach for a super-resolution mapping of satellite images, thereby instigating greater accuracy compared to a primary HNN model. GA is a computational processing algorithm inspired by Darwin's model, namely a survival for the fittest model (Feng et al., 2019). Furthermore,

Jayashree and Kumar (2019) had underlined mutation and crossover as the key traits of GA in order to extract information and prioritize feature selection. Consequently, it is one of the prevalent metaheuristics used by many neural networkers, substantiating its compatibility for a comparison with the Imperialist Competitive Algorithm (ICA) mechanism. The common ground in these works is the homogenization of the HNN framework with other evolutionary algorithms to enhance the HNN training stage in producing a better HNN mechanism. In order to introduce an all-rounded model, HNN can implement more vigorous metaheuristics in its training stage.

Generally, ICA is an evolutionary algorithm motivated by human socio-political behaviors. Imperialism is known as the practice of a government/country to grow stronger and rule beyond its territory, whereby the imperialist's main vision is to increase the number of colonies. The main components of ICA consist of the initial empires, assimilation, revolution, and imperialist competition (Li et al., 2019b). A work by Tashayo et al. (2019) had inaugurated its use to forecast the maximum surface settlement (MMS) induced by tunneling in civil projects. Meanwhile, Gerist and Maheri (2019) had proposed an approach to solve damage detection problems, specifically by utilizing ICA and resulting in a great performance of the convergence rate and better identification of the global optima both. Therefore, it can be concluded that employing ICA is an endless potential, ranging from industrial planning, scheduling, and decision-making to machine learning (Atashpaz-Gargari & Lucas, 2007). However, it is commonly used by researchers to explicitly acquire a solution to a problem, rather than making use of it to generate a learning model for the problem. A research by Abdechiri and Meybodi (2011) had emphasized the credibility of HNN in utilizing ICA to solve the propositional satisfiability (SAT) problem. However, the work is not suitable for solving real data sets. Therefore, in the current work, the proposed model employs ICA in the training stage to overcome the complexity of checking clause satisfaction, generate variation, and vast searching space in order to solve two real-life data sets acquired from UCI repository.

Due to the comprehensibility of the HNN framework, researchers have considered it as a black box model or a symbolic system. Taking this fact into consideration, the execution of logic learning in HNN has delineated many versatile models, primarily from the work of Abdullah (1992) that incorporates logic programming on HNN. The work presented an extensive HNN framework to cater for the optimization of logical consistency. Abdullah (1992) had accordingly proposed an optimized logic learning through synaptic weights, which was called the Abdullah (WA) method. In layman terms, logic programming illustrates the symbolic knowledge that will be "trained" by the HNN model. The primary work by Sathasivam (2012) had further fostered Abdullah's (1992) proposal by implementing first-order logic in a neuro-symbolic integration model, which attained more than 90% of global minimum energy (Kasihmuddin et al., 2019). Several

compelling logical rules such as the 2-Satisfiability (2-SAT) (Kasihmuddin et al., 2017a) and Maximum Satisfiability (MAX-*k*SAT) (Mansor et al., 2017) have been successfully embedded in HNN. Its application with propositional satisfiability logic is boundless, ranging from Very Large Scale Integration (VLSI) circuit configuration (Mansor et al., 2016) and Bezier curves satisfiability model (Kasihmuddin et al., 2016). For this work, the incorporation of the 3-Satisfiability (3-SAT) propositional logic is utilized due to its ability to achieve a higher probability of satisfied interpretation compared to Horn Satisfiability (Horn-SAT) and 2-SAT. Thus, the proposed HNN-ICA model is incorporated with 3-SAT in order to solve real-life applications.

Currently, no recent approach is available to thoroughly compare the performance of ICA with other metaheuristics in solving real-life data sets. This is crucial; as an evolutionary algorithm catering for variation and larger searching space in comparison with random search, ICA has to verify its distinctive features that can lead a better training model compared to other metaheuristics. Therefore, the contributions of this research are presented as follows: (1) to introduce the formulation of ICA with 3-SAT logic programming, (2) to initiate a model with the integration of HNN with ICA as a robust tool in order to solve optimization tasks by comparing it with two other searching techniques (i.e., GA and ES), (3) to implement reverse analysis with the proposed model of HNN-3SATICA in order to cater to real-life applications. The construction of the proposed model HNN-3SATICA shows better performance in the training stage and successfully interprets real-life datasets to detect the factors that are more prominent than others contributing to the optimization problems.

## MATERIALS AND METHOD

### 3-Satisfiability Logic (3-SAT)

Propositional satisfiability or SAT logic is perceived as a logical rule that consists of clauses containing literals or variables. General satisfiability logic (*k*-SAT) can signify the capability to represent real-life applications (Kasihmuddin et al., 2019). This work utilized discrete HNN which catered neurons in bipolar representation  $\{1, -1\}$  (Kasihmuddin & Sathasivam, 2016). Hansen et al. (2019) had emphasized on the generalization of *k*-SAT logical rule,  $P_{k-SAT}$  can be reduced to 3-SAT,  $P_{3-SAT}$  logical rule. The general formula of  $P_{3-SAT}$  is given as in Equation (1):

$$P_{3-SAT} = \bigwedge_{i=1}^m Z_i \tag{1}$$

where  $P_{3-SAT}$  is a 3-SAT that consists of clause  $Z_i$  shown in Equation (2):

$$Z_i = \bigvee_{j=1}^3 (x_{ij}, y_{ij}, z_{ij}) \tag{2}$$

whereby  $n$  literals and  $m$  clauses denoted by Conjunctive Normal Form (CNF) formula.

The general structure of  $P_{3-SAT}$  (Mansor et al., 2017) can be summarized as follows:

- i. A set of  $m$  clauses in a Boolean formula, where  $Z_1, Z_2, \dots, Z_m$  and clauses will be connected with logical AND operator  $\wedge$ .
- ii. Each clause consists of only literals will be combined by logical OR operator  $\vee$ .  
In the  $P_{3-SAT}$  formula, we only considered three literals in each clause.
- iii. Boolean Satisfiability formula composes an array of  $n$  literals,  $u_1, u_2, \dots, u_n$ , where  $u_i \in \{1, -1\}$  in each clause. Note that in this work,  $n$  is equal to 3.
- iv. The literals can be the variable itself or the negation of the variable, for example  $A$  or  $\neg A$ .

Further extension of  $P_{3-SAT}$ , an example of  $P_{3-SAT}$  is shown as follows:

$$P_{3-SAT} = (A \vee \neg B \vee C) \wedge (D \vee E \vee F) \wedge (\neg G \vee H \vee \neg I) \tag{3}$$

Equation (3) is satisfiable since it gives truth value resulting to  $P_{3-SAT}=1$ . According to Equation (3), if the neuron states read  $(A, B, C, D, E, F, G, H, I) = (-1, 1, 1, 1, 1, 1, 1, 1, -1)$ , the formula will be unsatisfiable or  $P_{3-SAT} = -1$ . In this research,  $P_{3-SAT}$  will be embedded to the proposed model, HNN-3SATICA in comparison with different learning algorithms.  $P_{3-SAT}$  will cater the modified networks to unveil the true pattern or behaviour of the real data sets involved. Note that  $P_{3-SAT}$  is a symbolic form representation thus it is appropriate to be integrated in these networks as HNN is a non-symbolic platform.

### Hopfield Neural Network (HNN)

HNN is a recurrent neural network, without hidden layer that mimics human biological brain. HNN structure of interconnected neurons and a powerful feature of CAM are crucial in solving various optimization and combinatorial tasks (Kong et al., 2019). The proposed model consists of structured  $N$  neurons, each of which is represented by an Ising variable. The neurons in discrete HNN are utilized in bipolar representation whereby  $S_i \in \{1, -1\}$  (Sathasivam, 2010). The fundamental overview for the bipolar neuron state activation in HNN is shown in Equation (4):

$$S_i = \begin{cases} 1 & , \text{ if } \sum_j W_{ij} S_j \geq \omega \\ -1 & , \text{ otherwise} \end{cases} \tag{4}$$

where  $W_{ij}$  refers to synaptic weight of the neuron from unit  $j$  to  $i$ .  $S_j$  is the state of neuron  $j$  and  $\omega$  is the predefined threshold value. Barra et al. (2018) specified that  $\omega = 0$  to verify the network's energy decreases and ascertain our network to achieve plausible results. The connection in Hopfield net contains no connection with itself  $W_{jj} = W_{ii} = 0$  (symmetrical). HNN model has similar intricate details to the Ising model of magnetism (Neelakanta & DeGroff, 1994). As the neuron states are termed in bipolar form, the neuron

rotates towards the magnetic field, resulting in the neurons to rotate until the equilibrium is achieved. Hence, the dynamic of HNN (considering all the neurons involved) is asynchronously changed according to  $S_i \rightarrow \text{sgn}[h_i(t)]$ , where  $h_i$  is the local field of the neurons connection. Motivated by Sathasivam et al. (2011), the sum of the field induced by each neuron is given in Equation (5):

$$h_i = \sum_{k=1, k \neq j}^N \sum_{j=1, j \neq k}^N W_{ijk} S_j S_k + \sum_{j=1, j \neq i}^N W_{ij} S_j + W_i \tag{5}$$

Thus, the biggest task of local field is to evaluate the final bipolar state of neurons and generate all possible  $P_{3-SAT}$  induced logic that was obtained from the final state of neurons. One of the most prominent features of HNN is the fact that it always converges in some cases, as illustrated by the following theorem (Hopfield, 1982).

**Theorem 1.** Let  $N$  be a neural network of order  $n$  and be defined by  $N = (W, T)$  where  $W$  is an  $n \times n$  matrix with element  $W_{ij}$  and  $T$  is a vector of dimension  $n$ , where element  $t$  depicted as the threshold attached to node  $i$ . The network will always converge to a stable state when running in serial mode; only one neuron can change the state at any time instantly, if the diagonal elements of  $W$  are non-negative.

Moreover, the subsequent state updating rule can be represented as in Equation (6)

$$S_i(t+1) = \text{sgn}[h_i(t)] \tag{6}$$

whereby **sgn** represents the signum function to squash the output of neurons, where this paper utilized Hyperbolic Tangent Activation Function (HTAF) (Mansor & Sathasivam, 2016). The following Equation (7) represents the Lyapunov energy function in HNN (Mansor et al., 2018a).

$$L_{P_{3-SAT}} = -\frac{1}{3} \sum_{i=1, i \neq j \neq k}^N \sum_{j=1, i \neq j \neq k}^N \sum_{k=1, i \neq j \neq k}^N W_{ijk} S_i S_j S_k - \frac{1}{2} \sum_{i=1, i \neq j}^N \sum_{j=1, i \neq j}^N W_{ij} S_i S_j - \sum_{i=1}^N W_i S_i \tag{7}$$

The energy value computed from the Equation (7) will be authenticated as global or local minimum energy. The network will provide the filtering mechanism and produce the correct solution when the induced neurons state reached global minimum energy. There are limited works to combine HNN and ICA as a single computational network. Thus, the robustness of ICA helps to improve training process in HNN.

### Imperialist Competitive Algorithm (ICA)

The pioneering work of ICA was presented by Atashpaz-Gargari and Lucas (2007), who stated that it was an algorithm inspired by imperialistic competition. Generally, all inspired countries are divided into two parts, namely the imperialist states and colonies, which tackle a list of operations such as initialization, assimilation, revolution, and imperialist

competition. This can lead to a better searching technique compared to other metaheuristics. The main purpose of ICA is to drive the colonies to converge to a global minimum solution, which is believed to have shown vigorous mechanisms in solving optimization tasks (Mollajan et al., 2019). It is set to be different compared to other metaheuristics as its features function to ease the performing neighborhood movements in both the continuous and discrete search spaces (Hosseini & Al Khaled, 2014). The application of ICA is infinitely many, such as ship design optimization (Peri, 2019), engineering design optimization (Aliniya & Keyvanpour, 2019), slope stability prediction (Koopialipoor et al., 2019), and heat and power dispatch problem (Davoodi & Babaei, 2019).

This work is focusing on utilizing ICA to find the maximum fitness of countries that will increase the number of satisfied clauses in the training stage. Its implementation with HNN is addressed as follows (Abdechiri & Meybodi, 2011):

**Step 1. Forming initial empires (Initialization)**

Each solution is shown by an array called country. Note that, in an  $N$ -dimensional optimization tasks, a country is denoted as  $1 \times N$  array. This array predefined as Equation (8):

$$C_{N_j} = \begin{cases} 1 & , \text{ rand}[0,1] \geq 0.5 \\ -1 & , \text{ otherwise} \end{cases} \tag{8}$$

whereby  $C_i$  is the country and  $C_{N_j}$  is the number of variables to be considered of interest about a country. Each empire  $E_i$  comprises  $N$  number of countries which represents the state of  $P_{3-SAT}$  as shown in Equation (9):

$$E_i = (C_1^{E_i}, C_2^{E_i}, \dots, C_N^{E_i}) \tag{9}$$

**Step 2. Fitness Evaluation**

Each country's fitness is calculated based on the clauses by using Equation (10):

$$f_{C_i^{E_i}} = \sum_{i=1}^{NZ} Z_i \tag{10}$$

$f_{C_i^{E_i}}$  denoted as fitness of each  $C_i$  in  $E^i$ ,  $Z_i$  is the clause in  $P_{3-SAT}$  and  $NZ$  is the total number of clauses.

**Step 3. Colonies moving towards imperialist (Assimilation)**

We select the imperialist,  $T^{E_i}$  from the country,  $C_i^{E_i}$  with the highest  $f_{C_i^{E_i}}$  and the rest will remain as colonies by using Equation (11).

$$C_i^{E_i} = \begin{cases} T^{E_i}, & \max \left[ f_{C_i^{E_i}} \right] \\ C_i^{E_i}, & \text{ otherwise, } i = \text{rand} [1, N] \end{cases} \tag{11}$$



Other remaining countries and colonies will be allocated to respective empires where each colonies population is randomized.

**Step 4. Revolution**

When revolution occurs, the rest of the colonies,  $C_i^{E_i}$  inside an empire  $E_i$  will be randomized according to the following Equation (12):

$$C_{ij}^{E_i} = \begin{cases} 1 & , \text{rand}[0,1] \geq 0.5 \\ -1 & , \text{otherwise} \end{cases} \tag{12}$$

to offer colonies acquiring better position that will attain higher chances of redeeming its place to take over the reigning empire by replacing the current imperialist,  $T^{E_i}$ . In order for revolution to take place, the fitness for each  $C_i^{E_i}$  will be computed by using Equation (10) and the new imperialist  $T^{E_i}$  will be selected based on Equation (11).

**Step 5. Imperialist Competition**

This feature of ICA sets apart ICA from other metaheuristics. In this process, imperialistic competition occurs among all imperialist in order to acquire power of each empires  $V_N^{E_i}$  as in Equation (13):

$$V_{E_i} = f_{T^{E_i}} - \varepsilon f_{T^{E_i}} + \frac{1}{N-1} \sum_{j=1}^N f_{C_j^{E_i}} \tag{13}$$

whereby  $f_{T^{E_i}}$  represents the fitness of each imperialist. According to Atashpaz-Gargari & Lucas (2007),  $\varepsilon = 0.05$  is chosen as an optimal value for this study. Empires that show no power will fall in this competition and the surviving imperialist will be the one that has the highest power. Worth mentioning that, if the power for a particular empire is within  $|V_{E_i} - V_{\phi}| \leq \lambda$ ,  $T^{E_i}$  will be chosen as the final neurons states for  $P_{3-SAT}$ . Note that  $\lambda$  will be predetermined by users. Step 4 and Step 5 will be repeated until termination criteria  $f_{T^{E_i}} = NZ$  is been met. The state of  $T^{E_i}$  will be stored as CAM. In this paper, we modified the ICA of the pioneering work (Atashpaz-Gargari & Lucas, 2007) that utilized ICA to solve a continuous problem into bipolar representation to solve NP problems. A work on implementing ICA with HNN for solving Satisfiability problem was executed by Abdechiri & Meybodi (2011), however it only catered to simulated data set. Figure 1 depicts the summary of ICA from Step 1 until Step 5. This research extends the work of ICA by implementing HNN and  $P_{3-SAT}$  logical rule with reverse analysis in solving real data sets.

**Genetic Algorithm (GA)**

In this paper, GA will be applied into HNN-3SAT or abbreviated as HNN-3SATGA in order to compare with other searching techniques, HNN-3SATICA and HNN-3SATES. GA is a popular optimization algorithm that was inspired by Darwin’s evolutionary theory to find a formula or optimized answer in order to predict or match patterns (Esfè et al., 2019).



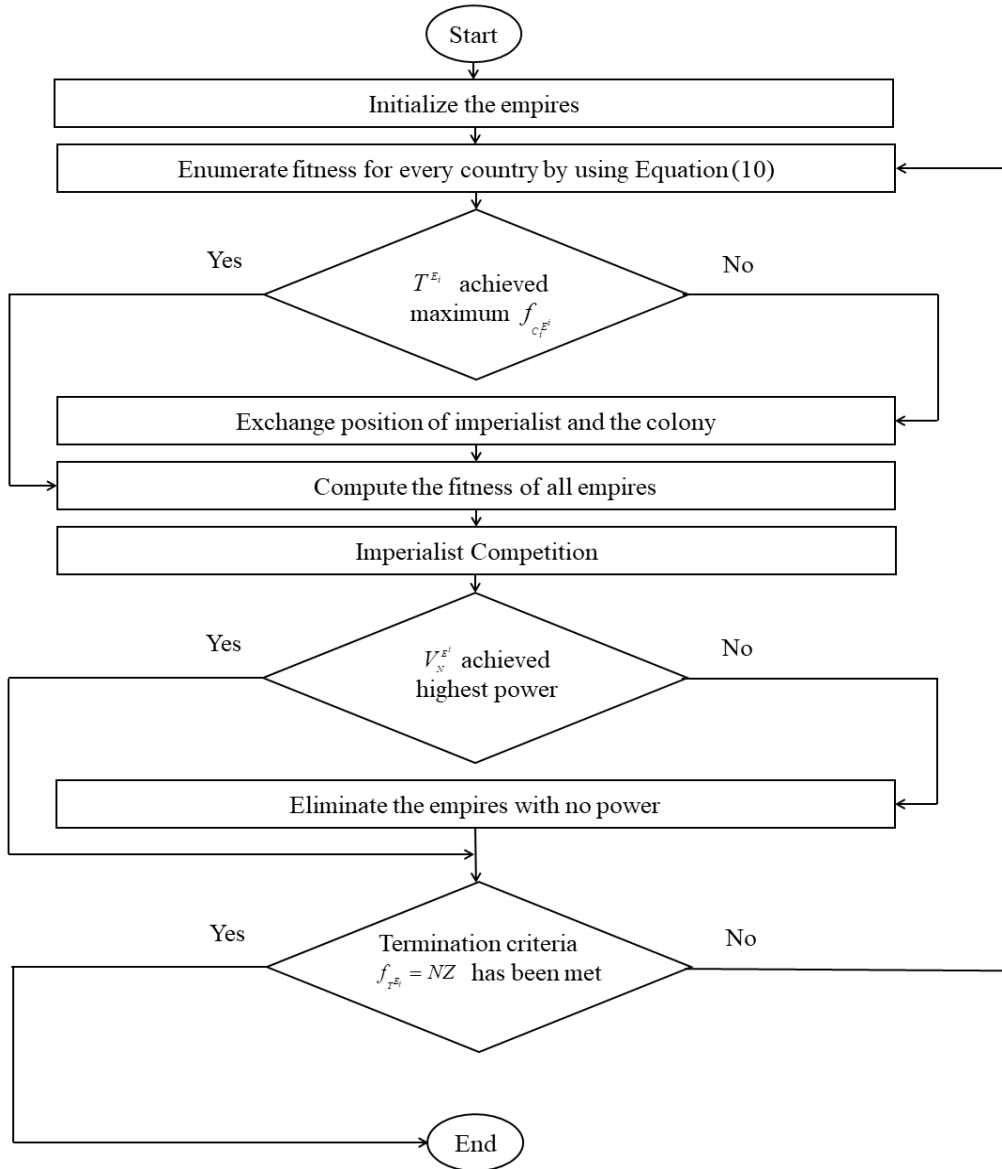


Figure 1. Summary of ICA

The phases of HNN-3SATGA are as follows (Mansor et al., 2019):

**Step 1. Initialization**

Initialize 100 random chromosomes ( $Cr_i$ ) as bipolar interpretation where each element of  $\{1, -1\}$  is denoted by True and False. Every  $Cr_i$  portrays the possible interpretation for  $P_{3-SAT}$ .

**Step 2.  $Cr_i$  Fitness Computation**

Fitness of the  $Cr_i$  in GA is typically being calculated by using fitness function. Thus, the fitness computation is based on the satisfiability of the clauses in  $P_{3-SAT}$ . Hence, the maximum fitness manifests the effectiveness of the training process.

**Step 3.  $Cr_i$  Selection**

Select  $y$   $Cr_i$  (in our case  $y = 10$ ) containing  $P_{3-SAT}$  information with the highest fitness out of 100  $Cr_i$  to undergo the crossover process.

**Step 4. Crossover**

Two  $Cr_i$  are selected and separated during the crossover phase from 10 selected  $Cr_i$ . Furthermore, by joining both parts of the paternal  $Cr_i$ , a child  $Cr_i$  is produced. Hence, the second child is therefore generated by the addition of the primary and secondary segment and vice versa (Luke, 2013).

For example:

**Before crossover**

$$Cr_1 = -1-1-1111-1-1-1$$

$$Cr_2 = -1-1-11-11-1-1-1$$

**After crossover**

$$Cr_1 = -1-1-1-1-1-1-1-1-1$$

$$Cr_2 = -1-1-11-1111$$

Therefore, the  $Cr_i$  fitness for the new generated  $Cr_i$  can be determined.

**Step 5. Mutation**

Mutation is an integral optimization operator in the GA which shifts  $Cr_i$  patterns, ensuring that the population is not trapped at local minima. This is the random process of altering specific genes of the  $Cr_i$ . As an illustration:

**Before mutation**

$$Cr_3 = -1-1-11-1111$$

**After mutation**

$$Cr_3 = 1-1-11-1111$$

The first part of the  $Cr_i$  was flipped from -1 to 1. Thus, a better  $Cr_i$  might be generated after mutation. The fitness value for a newly formed  $Cr_i$  will be computed. The current  $Cr_i$  will repeat the first step if the fitness value does not achieve maximum fitness.

## IMPLEMENTATION

### Performance Evaluation Metrics

The performance of the proposed model in executing logic mining will implement several performance metrics such as root mean square error (RMSE), mean absolute error (MAE), global minima ratio ( $R_m$ ), computational time ( $CT$ ) and accuracy ( $Q$ ). The list of parameter names in performance evaluation metrics is shown in Table 1. Meanwhile, the list of parameter values used in  $R_m$  depicts in Table 2.

### Root Mean Square Error (RMSE)

Overall, RMSE is a prediction metric utilized by neural networkers to enumerate the predicted value of a model with observed value. RMSE formulation utilized in this work is shown in Equation (14):

$$RMSE = \sum_{i=1}^n \sqrt{\frac{1}{n} (f_x - f_y)^2} \quad (14)$$

### Mean Absolute Error (MAE)

According to Willmott and Matsuura (2005), MAE capable of evaluating good error estimation by showing a uniformly distributed error. MAE formulation consists of the absolute value of the difference between the estimated values and the actual values (Chai & Draxler, 2014). A good model will attain low values of RMSE and MAE. The equation of MAE is shown in Equation (15):

$$MAE = \sum_{i=1}^n \frac{1}{n} |f_x - f_y| \quad (15)$$

### Global Minima Ratio ( $R_m$ )

The global minimum ratio is the generalized metric for evaluating the efficiency of the solutions. Tolerance value will filter each calculated final energy of the neurons in HNN. The final energy is assumed as global minimum energy if the final energy of the model within the tolerance value (Sathasivam, 2010). The Equation (16) of  $R_m$  is shown as follows:

$$R_m = \frac{1}{ab} \sum_i^n N_{L_{P3-SAT}} \quad (16)$$

### Computational Time ( $CT$ )

Computational time is used to determine the effectiveness of the proposed models. The

value for  $CT$  will be measured in SI unit of second ( $s$ ). The Equation (17) of  $CT$  is shown below:

$$CT = TrainingTime(s) + TestingTime(s) \tag{17}$$

whereby  $TrainingTime$  and  $TestingTime$  are depicted as the total time to execute the HNN-3SAT models in training and testing phase respectively. In the work by Kho et al. (2020),  $CT$  was utilized for HNN-2SAT model because it implies the capability and stability of the model.

**Accuracy ( $Q$ )**

The accuracy is used to assess the models ability to train the data set. The Equation (18) of  $Q$  is shown below:

$$Q = \frac{P_{induced}^{Correct}}{N_{P_{test}}} \times 100\% \tag{18}$$

Table 1  
List of parameters in performance evaluation metrics

Parameter	Parameter name	Parameter	Parameter name
$f_x$	Total number of clauses	$N_{L_{P3-SAT}}$	Number of global minimum energy
$f_y$	Number of satisfied clause	$N_{L_{P3-SAT}}$	Number of global minimum energy
$n$	Number of iteration before $f_x = f_y$	$N_{P_{test}}$	Number of testing data
$a$	Number of trials	$P_{induced}^{Correct}$	Correct induced logic
$b$	Number of neuron combination		

Table 2  
List of parameters in  $R_m$

Parameter	Parameter value
$a$	100
$b$	100
Tolerance value	0.001 (Sathasivam, 2010)

### Experimental Setup

The utilization of  $P_{3-SAT}$  will aid in discovering valuable information on real data sets. Many works of literature have surfaced in explaining that information and patterns can be represented in a logical form. The use of reverse analysis method to extract significant information from a particular data set has been introduced by Sathasivam and Abdullah (2011) by considering the CNF logical rule. Motivated by the work of Mansor et al. (2018b), more systematic logic mining techniques incorporating  $P_{3-SAT}$  in HNN have been proposed. Following this, the 3-SAT-based Reverse Analysis Method (3-SATRA) is employed in HNN-3SATICA to generate an optimized induced logical rule from several prominent data sets. In this case, raw data are translated into  $P_{3-SAT}$  and then embedded and processed by HNN-3SAT. By pursuing this, the induced  $P_{3-SAT}$  will be used to classify the outcome of the dataset. The comprehensive 3-SATRA via HNN-3SATICA is depicted in Figure 2.

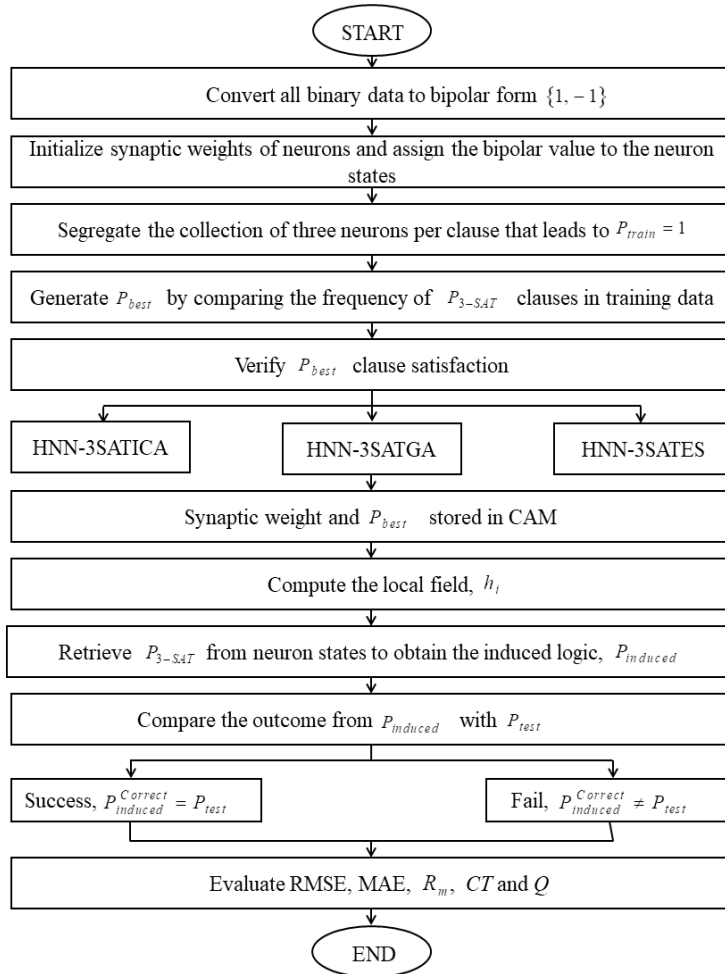


Figure 2. Implementation of 3-SATRA in HNN-3SATICA

In this paper, two different real data sets from different fields occupied the constructed 3-SATRA, namely the Bach Choral Harmony data set (BCHDS) and German Credit data set (GCDS). All data sets were taken from the UCI machine learning repository website and each of them had different purposes. Nine attributes were used in this paper and all real data sets used multivariate data. In this experiment, the aim was to analyze a comprehensive comparison of accuracy between HNN-3SATICA and other existing models such as GA, ES, and researchers utilizing the same data sets. The information about BCHDS and GCDS is shown in Table 3 and 4.

Table 3

List of attributes for each data sets

Data Set	Details of each attributes	Output $P_{3-SAT}$
BCHDS (Haque et al., 2019)	$A$ : Pitch for Key “C-M”	To identify the distinction between good harmony and bad harmony for musician.
	$B$ : Pitch for Key “E-m”	
	$C$ : Pitch for Key “B-m”	
	$D$ : Pitch for Key “A-m”	
	$E$ : Pitch for Key “F-m”	
	$F$ : Pitch for Key “G-M”	
	$G$ : Pitch for Key “A-M”	
	$H$ : Bass	
	$I$ : Meter	
GCDS (Liu et al., 2019)	$A$ : Credit History	To distinguish bank’s customer of having a good or bad credit risks.
	$B$ : Status of existing checking account	
	$C$ : Saving account/bonds	
	$D$ : Personal status	
	$E$ : Age	
	$F$ : Housing	
	$G$ : Number of existing credits at this bank	
	$H$ : Telephone	
	$I$ : Foreign worker	

As mentioned earlier, the literals can be the variable itself or the negation of the variable; for example,  $A$  or  $\neg A$ . Thus, as the work implemented real-life data sets, the negation literals represented attributes that did not affect the output of  $P_{3-SAT}$  and *vice versa* for non-negate literals.

Table 4

*Method used in existing model*

Data Set	Method
BCHDS	AdaBoost
GCDS	Support Machine Vector (SVM)

The source code was developed through Dev C++ Version 5.11 in 8GB RAM with Intel Core i5 to acquire a good comparison between all proposed models. The same device was used during simulation to avoid bias; using the same device rendered the simulation comparable as the memory (RAM) and processor had the same power. Another factor that can trigger bias is a different compiler for each learning algorithm, which may lead to different computational times or anomaly in error values. Moreover, to avoid bias, the number of neurons chosen for the simulations showed the same combination of  $9 \leq NN \leq 72$  for all learning algorithms. The threshold for simulation was set to 24 hours according to Kasihmuddin et al. (2016). All outputs exceeding 24 hours of computation time were omitted due to the simulation, which would eventually break down in finding the satisfied clause interpretation,  $E_{P_{3-SAT}} = 0$ . Table 5, 6 and 7 show all parameters involved in the proposed models.

Table 5

*List of parameters in HNN-3SATES (Sathasivam, 2012)*

Parameter	Parameter value
Neuron Combination ( $b$ )	100
Number of Trial ( $a$ )	100
Tolerance Value ( $Tol$ )	0.001
Number of String	100
Selection Rate	0.1

Table 6

*List of parameters in HNN-3SATGA (Kasihmuddin et al., 2017b)*

Parameter	Parameter value
Neuron Combination ( $b$ )	100
Number of Trial ( $a$ )	100
Tolerance Value ( $Tol$ )	0.001
Selection Rate	0.1
Mutation Rate	0.01
Generation	1000



Table 7

*List of parameters in HNN-3SATICA*

Parameter	Parameter value
Neuron Combination ( $b$ )	100
Number of Trial ( $a$ )	100
Tolerance Value ( $Tol$ )	0.001
Initial Empires	10
Parameter ( $\epsilon$ )	0.05
Termination Value ( $\lambda$ )	0.05

## RESULT AND DISCUSSION

The effectiveness of implementing ICA in the training stage of the HNN-3SAT model was investigated in this paper. The proposed mode of HNN-3SATICA in comparison with existing models that utilized GA (Kasihmuddin et al., 2017b) and ES (Sathasivam, 2012) embedded two real-life data sets, which were retrieved from the UCI Machine Learning Repository platform. The investigation of a model's performance can be separated into two parts. The first important part is to examine the quality of the solution generated by different searching techniques, specifically by employing the suitable training errors. Second, one should analyse the robustness and efficiency of the proposed model by comparing the  $CT$  and  $Q$  needed to execute the models' respective mechanisms. Accordingly, five performance evaluation metrics were involved in analyzing the training and testing stages of the modified models, as presented in the Performance Evaluation section. Therefore, this research's main contribution was the display of HNN-3SATICA competency in outperforming the existing models.

In this section of result analysis for BCHDS, it could be concluded that the outcomes attained by HNN-3SATICA for RMSE and MAE showed a consistent value of zero (Figure 3 & 4), thereby indicating ICA providing a better and well-trained HNN framework. However, the results were identical to HNN-3SATGA. Similarly, HNN-3SATES projected larger errors as the number of neurons increased. The incorporation of ES underlined the lack of modification undertaken in the training stage of the fundamental HNN. For the  $R_m$ , it is likely to be a better model when it is prone to the value of 1 (Sathasivam, 2011). From Figure 5, all models are approaching to 1 even by manipulating different numbers of neurons. The trend showed the capability of the training methods deployed by HNN-3SAT to attain the global minimum energy by having  $R_m$  closer to 1. From Figure 6, it can be deduced that all models exhibit less  $CT$  compared to the assigned threshold time. Regardless, HNN-3SATICA executed less  $CT$  compared to other models, thereby displaying

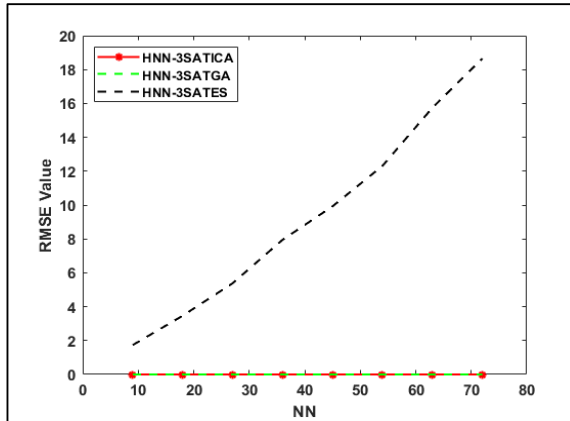


Figure 3. RMSE value of HNN-3SAT models for BCHDS

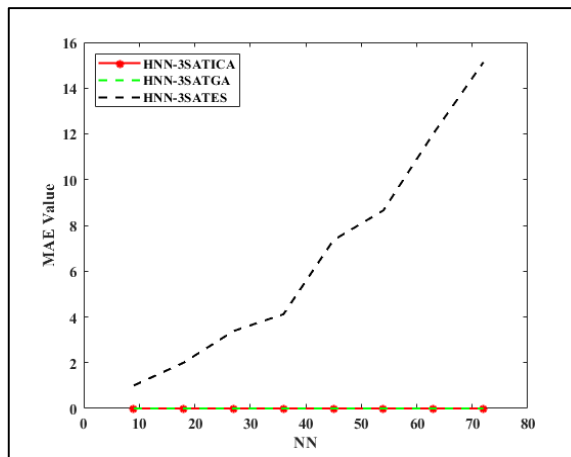


Figure 4. MAE value of HNN-3SAT models for BCHDS

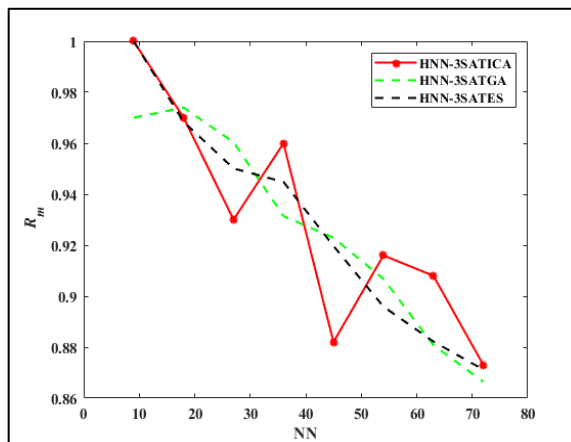


Figure 5.  $R_m$  of HNN-3SAT models for BCHDS

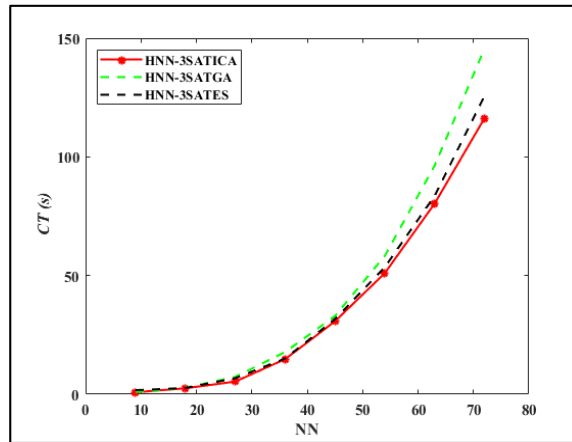


Figure 6. CT of HNN-3SAT models for BCHDS

Table 8

Accuracy of HNN-3SATICA with other existing models

Data Set	HNN-3SATICA	Sathasivam (2012)	Kasihmuddin et al. (2017b)	Haque et al. (2019)
BCHDS	61%	54%	61%	56.72%

the effectiveness of ICA during the training stage with different levels of complexity. Table 8 outlines the accuracy of all models that utilized BCHDS, whereby HNN-3SATICA and HNN-3SATGA are observed to enumerate the same value of accuracy. Both models achieved 61% of accuracy, which demonstrated their respective capability in attaining an optimized induced logic for BCHDS. Besides, HNN-3SATES and the work by Haque et al. (2019) both generate an accuracy value less than 60%. The accuracy for both methods was not promising, particularly for HNN-3SATES. It is due to the non-optimized induced logic generated at the end of the executions. By executing the simulation of HNN-3SATICA for BCHDS, the induced logic attained by HNN-3SATICA can be observed as per Equation (19) below:

$$P_{induced}^{Correct} = (\neg A \vee B \vee \neg C) \wedge (D \vee E \vee \neg F) \wedge (G \vee H \vee I) \tag{19}$$

whereby from Equation (19), in order to produce a good harmony, attributes  $A$ ,  $C$ , and  $F$  are insignificant to be scrutinize. However, such attributes like  $B$ ,  $D$ ,  $E$ ,  $G$ ,  $H$  and  $I$  would lead the harmonization to be out of tune.

In comparison with the result analysis of BCHDS, GCDS showed better result of HNN-3SATICA with other existing models. Figure 7 reveals the RMSE values attained by HNN-3SATGA and HNN-3SATES as relatively increasing as the number of neurons increased. However, HNN-3SATICA persistently achieved zero RMSE values despite the

incremental number of neurons. Furthermore, the MAE analysis for all models yielded comparatively similar outcomes to the illustration of RMSE analysis. From Figures 7 and 8, the execution of the ICA mechanism is significant to aid the standalone HNN with a vigorous training stage to achieve  $E_{P_{3-SAT}} = 0$ . Unfortunately, both HNN-3SATGA and HNN-3SATES revealed an inclining trend of errors for their MAE and RMSE outcomes both, showing the incompetency of the GA and ES mechanisms to accommodate a higher number of neurons as the complexity increased. The analysis of  $R_m$  in GCDS (Figure 9) shows that most of the models are able to generate at most 100% of global minimum solutions, except for HNN-3SATGA. Although two different data sets from different fields were utilized, the findings of  $R_m$  achieved by all models depicted indistinguishable results. Figure 10 displays that the  $CT$  for HNN-3SATGA and HNN-3SATES requires more time compared to HNN-3SATICA. This was due to the proposed model offering a larger search space, which contributed to a more efficient HNN framework. In Table 9, the induced logic generated by HNN-3SATICA in the testing stage records an accuracy of 83%. This finding set forth its ability to acquire an optimized induced logic that could best represent the GCDS data set. Contrary to this, HNN-3SATGA, HNN-3SATES, and SVM methods deployed by Liu et al. (2019) have achieved lower accuracy compared to HNN-3SATICA. This is attributable to ICA mechanism's function to attain  $E_{P_{3-SAT}} = 0$ , which can lead to a better training stage in the resulting construction of an optimized induced logic. Equation (20) displays the induced logic attained by HNN-3SATICA at  $NN = 54$  until  $NN = 72$ :

$$P_{induced}^{Correct} = (\neg A \vee B \vee \neg C) \wedge (D \vee E \vee \neg F) \wedge (G \vee H \vee I) \quad (20)$$

From Equation (20), one can distinguish whether a customer is a good credit risk or not, whereby attributes such as  $A$  and  $D$  can exhibit a fair credit status. Here, the credit history and personal status were very important to know their credit management and liability. Other than that, the induced logic could reveal insignificant and trivial attributes such as  $C$  and  $I$  in order to sort out which customer was a good or bad credit risk.

The training stage played a prominent role in enhancing the standalone HNN framework. From the HNN-3SATICA results obtained in Figure 3 and 4 and Figure 7 and 8, the capability of ICA is portrayed by improving the HNN training stage to attain a good solution. The deviation of error was generally smaller than the other counterparts due to the optimization operator employed in ICA. Assimilation and revolution in ICA played a big role to generate fewer iterations in obtaining  $E_{P_{3-SAT}} = 0$ . Apart from this, the fewer iterations indicated that less RMSE and MAE were generated from the model. However, the results attained for HNN-3SATGA and HNN-3SATES were larger than the proposed model, particularly in GCDS. They both employed undeniably ineffective searching method compared to ICA, especially ES. In particular, ES used random search where the number of neurons increases, the complexity also increases. Thus, ES contributes to

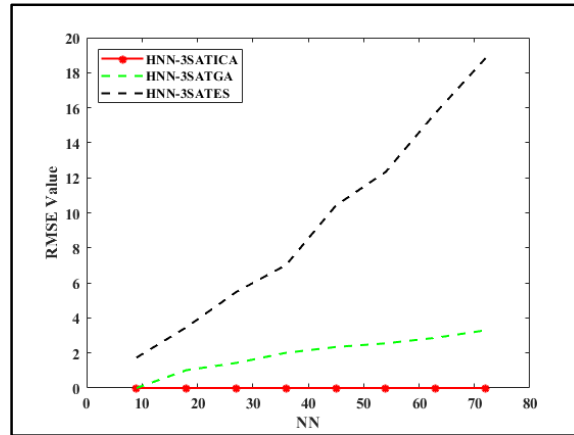


Figure 7. RMSE value of HNN-3SAT models for GCDS

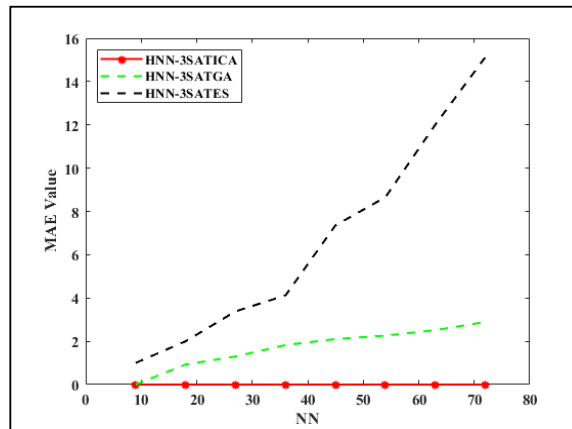


Figure 8. MAE value of HNN-3SAT models for GCDS

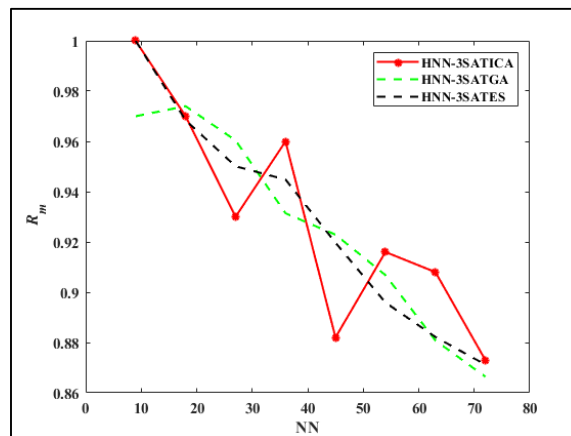


Figure 9.  $R_m$  of HNN-3SAT models for CGDS

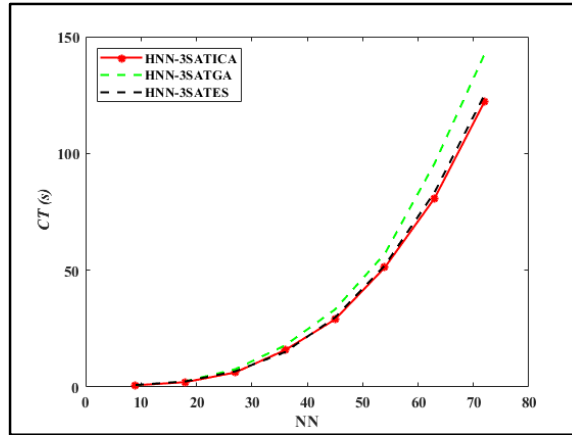


Figure 10. CT of HNN-3SAT models for BCHDS

Table 9

Accuracy of HNN-3SATICA with other existing models

Data Set	HNN-3SATICA	Sathasivam (2012)	Kasihmuddin et al. (2017b)	Liu et al. (2019)
GCDS	83%	71%	82%	75.7%

generate larger errors. Figures 5 and 9 showcase the  $R_m$  obtained by all models, which are relatively identical even as the number of neurons increased. Therefore, this ensured the proposed model reached the global minimum energy without any complications. This fact displayed the versatility and relevancy of the ICA mechanism to accomplish close achievement as the existing models.

$CT$  is predefined as the expanse of time needed for the network to complete the overall computational process. Therefore, it is significant to identify a model’s performance efficiency affirmation (Xiao et al., 2017). From Figures 6 and 10, it is discovered that  $CT$  needed to execute HNN-3SATICA is lesser than other existing models. In contrast, with HNN-3SATES, HNN-3SATICA attained faster execution due to the ICA algorithm that had fewer parameters compared to ES. However, for ES, the number of neurons was gradually increasing and thus more  $CT$  was required due to its nature of brute force that needed more iterations. Thereby resulting in overfitting the solution. HNN-3SATICA was set apart from other models as it achieved a satisfactory percentage of accuracy for both BCHDS and GSDS data sets. Based on the results in Table 9, it can be concluded that ICA plays a vital role in the proposed model to generate better-induced logic, which reflects the precision of the modified network. The reason for ICA being able to fulfill such factor is due to the optimization operator in it, which introduces a larger search space and variations of solution that contribute to an effective training stage.

The limitation of HNN-3SATICA is that this modified framework only caters to bipolar representation and only utilizes the multivariate type of data sets. Further extension can be done by utilizing the binary form of entries and incorporating other types of data sets, such as time-series. Other than that, this modified network only utilized one type of SAT, namely 3-SAT. Therefore, one can incorporate more than one or other types of SAT, such as 2-SAT, MAX- $k$ SAT, and  $k$ -SAT. The main drawback of HNN-3SATICA is its tendency of overfitting, which lacks the variability of generated induced logic. In the current context, overfitting indicates that as the search algorithm becomes more complex in the training phase, the solution will produce inaccurate and biased results (Reunanen, 2003). To overcome such aspect, an alteration of the data sets is crucial. Rearrangement and permutation of the attributes with a randomized selection of attributes should be implemented. The no-free lunch theorem (Wolpert & Macready, 1997) states that there are no absolute or specific algorithms that can be utilized to solve any problems. However, in this research, it was found that HNN-3SATICA worked exceptionally well for GCDS. On the contrary, the HNN-3SATICA and HNN-3SATGA worked well for BCHDS.

## CONCLUSION

In this research, the formulation of constructing ICA in 3-SAT proved to be adequate to represent the mechanism that involves in ICA and the implementation of ICA with standalone HNN framework with reverse analysis proved to be effective in solving real life data sets. Through this approach, we have successfully presented a modified model of HNN-3SATICA which presented ICA role in generating variations and broad searching space. Particularly in this research, assimilation and revolution components of ICA provided a better solution in checking  $E_{P_{3-SAT}} = 0$  which modify the training stage of a fundamental HNN framework. The proposed HNN-3SATICA model was trained and tested by using two real life data sets, in comparison with two different searching techniques; HNN-3SATGA and HNN-3SATES. The performance evaluation between these models was analysed by employing different performance metrics such as RMSE, MAE, global minima ratio, computational time and accuracy. The analysis of the results displayed the competency of the proposed model, HNN-3SATICA for all chosen performance evaluation metrics. We have successfully accomplished all objectives as presented based on the performance of our proposed model. Computational simulations for all models were specifically difficult towards larger number of neurons. However, the constructed proposed model, HNN-3SATICA showed good potential and a better training mechanism compared other metaheuristics and regular HNN framework. Extended research is required to further enhance HNN-3SATICA model by employing the same mechanism to other types of recurrent neural networks, such as Elman and Kohonen neural network. Other than that, we can utilize other types of propositional logic such as  $k$ -SAT and MAX- $k$ SAT.



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